STAT 514 Midterm 2 Solution (Total 50 Points)

1. (10 points) Consider a Latin square design with 3 treatments. Assume MSE=18 and we would like to test the contrast: $\mu_1 - 2\mu_2 + \mu_3 = 0$.

   (1) What are the degrees of freedom for the overall F-test?
   Numerator d.f. = p – 1 = 3 – 1 = 2
   Denominator d.f. = (p – 2) (p – 1) = (3 – 2) (3 – 1) = 2

   (2) What is the standard error of the overall mean?

   $$\text{var}(\bar{y}) = var(\frac{\sum y_i}{p^2}) = \frac{\sum \text{var}(y_i)}{p^4} = \frac{p^2\sigma^2}{p^4} = \frac{\sigma^2}{p^2}$$ *, so*

   $$SE(\bar{y}) = \sqrt{\frac{MSE}{p^2}} = \frac{\sqrt{18}}{3} = \sqrt{2}$$

   (3) What is the degree of freedom for a t-test on the contrast?
   d.f. = 2

   (4) What is the standard error of the estimate of the contrast?

   $$SE_C = \sqrt{MSE\sum\frac{C_i^2}{n_i}} = \sqrt{18\times(\frac{1}{3}+\frac{4}{3}+\frac{1}{3})} = 6$$

2. (20 points) Disk drive substrates may affect the amplitude of the signal obtained during readback. A manufacturer compares four substrates: aluminum(A), nickel-plated aluminum(B), and two types of glass(C and D). Sixteen disk drives will be made, four using each of the substrates. The design responses (in microvolts) are given in the following table (data from Nelson 1993, Greek letters indicate day):

Machine	Operator			
	1	2	3	4
1	Aα = 8	Cγ = 11	Dδ = 2	Bβ = 8
2	Cδ = 7	Aβ = 5	Bα = 2	Dγ = 4
3	Dβ = 3	Bδ = 9	Aγ = 7	Cα = 9
4	Bγ = 4	Dα = 5	Cβ = 9	Aδ = 3

   The grand mean is 6, and the level means for the four substrates are

   A: 5.75 B: 5.75 C: 9.00 D: 3.50

   (1) What kind of design is used for the experiment? Describe the major advantages.
   The design is Graco-Latin Square.
   This design controls 3 sources of extraneous variability (blocks in 3 directions).

(2) Calculate the estimates of the treatment effects.

Overall mean = 6

$$\hat{\tau}_A = 5.75 - 6 = -0.25$$
$$\hat{\tau}_B = 5.75 - 6 = -0.25$$
$$\hat{\tau}_C = 9.0 - 6 = 3.0$$
$$\hat{\tau}_D = 3.50 - 6 = -2.50$$

(3) Complete the following output from SAS.

```
Source                    DF        Squares     Mean Square    F Value    Pr > F

Model                     12     100.5000000      8.3750000       1.17    0.5098

Error                      3      21.5000000      7.1666667

Corrected Total           15     122.0000000


Source                    DF       Type I SS    Mean Square    F Value    Pr > F

row                     ?? 3     21.50000000     7.16666667       1.00    0.5000
col                     ?? 3     14.00000000     4.66666667       0.65    0.6335
substrate               ?? 3     ?? 61.5        ??20.5          ??2.86 ??0.1<p<0.25
greek                   ?? 3      3.50000000     1.16666667       0.16    0.9149
```

Test if the substrates are different in terms of their effects on the responses at α = 0.1. State the hypotheses, obtain the test statistic and draw your conclusion.

Answer 1:

$$H_0 : \tau_A = \tau_B = \tau_C = \tau_D = 0 \quad \text{vs} \quad H_1 : \text{at least one } \tau_i \neq 0$$

$$F = \frac{SS_{substrate}/3}{MSE} = \frac{61.5/3}{7.1666667} = \frac{20.5}{7.1666667} = 2.8605$$

$$F_{0.1,3,3} = 5.39$$

Since $2.8605 < F_{0.1,3,3}$, fail to reject H_0.

(4) If the machine were not considered as blocks and not included in ANOVA model, will the test results change? Justify your answer.

Then $MSE = \dfrac{21.5 + 21.5}{3+3} = 7.1666667$

So there will be no change.

3. (20 points) A chemical production process consists of a first reaction with an alcohol and a second reaction with a base. A factorial experiment with three alcohols and two bases was conducted with three replicate reactions conducted in a completely randomized design. The collected data were percent yield.

Base	Alcohol

	1	2	3
1	91, 90, 91	89, 88, 90	87, 88, 90
2	87, 88, 91	91, 92, 95	90, 92, 93

Here are some summary statistics:

Grand mean: 90.1667

Base	mean
1	89.3333
2	91.0000

Alcohol	mean
1	89.6667
2	90.8333
3	90.1667

Base	Alcohol	mean
1	1	90.6667
1	2	89.0000
1	3	88.3333
2	1	88.6667
2	2	92.6667
2	3	91.6667

(1) Write a linear model for this experiment, explain the terms and specify assumptions.

$$y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \varepsilon_{ijk},\ \varepsilon_{ijk} \sim iid\ \ N(0, \sigma^2)$$

$$i = 1, 2;\ \ j = 1, 2, 3;\ \ k = 1, 2, 3$$

(2) What are the constraints need to be satisfied?

$$\sum \tau_i = 0,\ \sum \beta_j = 0,\ \sum_i (\tau\beta)_{ij} = 0,\ \sum_j (\tau\beta)_{ij} = 0$$

(3) What are the estimates of effects for base = 1, and for the following combination of base and alcohol: (base, alcohol) = (2,3)?

$$\hat{\tau}_1 = 89.3333 - 90.1667 = -0.8334$$

$$\widehat{(\tau\beta)}_{2,3} = \bar{y}_{23\cdot} - \bar{y}_{2\cdot\cdot} - \bar{y}_{\cdot 3\cdot} + \bar{y}_{\cdots} = 91.1667 - 91 - 90.1667 + 90.1667 = 0.6667$$

(4) Complete the following ANOVA table from SAS, and test if the main effects and interactions are significant at α = 0.05.

Source	DF	Squares	Mean Square	F Value	Pr > F
Model	5	47.16666667	9.43333333	3.86	0.0257
Error	12	29.33333333	2.44444444		
Corrected Total	17	76.50000000			

Source	DF	Type I SS	Mean Square	F Value	Pr > F
base	1	??12.500500	??12.500500	??5.1138	??<0.05
alcohol	2	4.33333333	??2.166667	??0.8864	??>0.25
base*alcohol	2	??30.3328333	??15.16641665	??6.2044	??<0.025

$$SS_{base} = bn\sum_i (\bar{y}_i.. - \bar{y}...)^2$$

$$= 3 \times 3[(89.3333 - 90.1667)^2 + (91 - 90.1667)^2] = 12.500500$$

$$F_{0.05,1,12} = 4.75, \text{ so } p_{base} < 0.05$$

$$F_{0.25,2,12} = 1.56, \text{ so } p_{alcohol} > 0.25$$

$$F_{0.025,2,12} = 5.1, \text{ so } p_{base*alcohol} < 0.025$$

(5) Use Bonferroni method to compare the following treatments: (1, 1), (1, 2), (1, 3), and (2, 1) pairwisely, use α = 0.05. Calculate the critical difference and report the results.

$$CD = t_{\alpha/2m,ab(n-1)}\sqrt{MSE\frac{2}{n}} = t_{0.05/(2\times6),2\times3\times(3-1)}\sqrt{2.4444\times\frac{2}{3}} \cong 3.899884$$

$$t_{0.05/(2\times6),2\times3\times(3-1)} \cong t_{0.005,12} = 3.055$$

None of the pairwise differences is significant.

(6) Ignore the interaction between base and alcohol, and suppose Tukey's method is used to compare the level means of alcohol pairwisely. Calculate the critical difference use α = 0.05.

$$CD = \frac{q_\alpha(b, ab(n-1))}{\sqrt{2}}\sqrt{MSE\frac{2}{na}} = \frac{q_{0.05}(3,12)}{\sqrt{2}}\sqrt{2.4444\times\frac{2}{3\times2}} = \frac{3.77}{\sqrt{2}}\sqrt{2.4444\times\frac{2}{3\times2}}$$

$$= 2.4063$$

(7) Interpret the following interaction plot between base and alcohol.

The plot shows interaction exists.